FORCE ANALYSIS OF MECHANISMS

Mechanisms and machines are dynamic systems that are designed to perform some specific task or function. A mechanism performs some kind of work on these external forces or torques and there are also external forces and torques that are required to make the mechanism move. In addition to the external forces, inertial forces are developed within the mechanism due to the acceleration and deceleration of the various members of the mechanism. All of these forces are transmitted from one link to the next through the connections between the links. A dynamic force analysis of the mechanism must be performed in order to determine the forces that are transmitted by the joints and the torque or force required to make the mechanism operate at the design speed. The actual joints can be designed once the forces that are transmitted through the joint have been determined.

DYNAMICS FUNDAMENTALS

Newton's Laws of Motion establish the foundation for the study of dynamics. These laws are summarized as follows:

1. A body at rest remains at rest and a body moving with constant velocity remains at constant velocity unless acted on by an external force.

2. The time rate of change of the momentum is equal to the applied force and acts in the direction of the applied force.

$$\bar{F} = \frac{d(m\bar{v})}{dt} = m\bar{a} \qquad\qquad \bar{T} = \frac{d(I\dot{\theta})}{dt} = I\ddot{\theta}$$

3. For every action there is a reaction that is equal in magnitude and acts in the opposite direction.

$$\bar{F}_{\Theta\theta} = -\bar{F}_{\theta\Theta} \qquad\qquad \bar{T}_{\Theta\theta} = -\bar{T}_{\theta\Theta}$$

You learned to use these laws to determine the motion of rigid bodies in your course on dynamics. The problems you encountered typically required you to determine the resulting motion of a body or system given the external forces acting on the system. This type of problem, in which the forces are known and the equations of motion are to be determined, is called a forward dynamics problem. In the forward dynamics problem you must solve the differential equations of motion. The type of problem we encounter in mechanism design is the opposite of the forward dynamics problem. We know or specify the required motion of the mechanism. From a kinematic analysis of the mechanism we can determine the position, velocity and acceleration of all elements of our mechanism. What we would like to determine are the forces and torques that develop as a consequence of the desired motions. This type of problem, wherein the motion is known and the forces are to be determined, is called the inverse dynamics problem. It does not require us to solve the differential equations of motion since we already know what the motions are. We simply apply the equations of equilibrium to the elements of our system in order to determine the forces that develop.

It is common practice in dynamics to replace the physical system we are analyzing with an equivalent dynamic system that has the same dynamic response as our physical system. In our dynamic model, we replace the physical members of our mechanism with point masses connected by massless, rigid links. We will concentrate the mass of our link at the center of gravity of the link and we will also compute the mass moment of inertia of our physical link with respect to an axis passing through the center of gravity of the link. We can use the parallel axis theorem to calculate the mass moment of inertia with respect to an axis passing through a point other than the center of gravity.

A SINGLE LINK IN PURE ROTATION

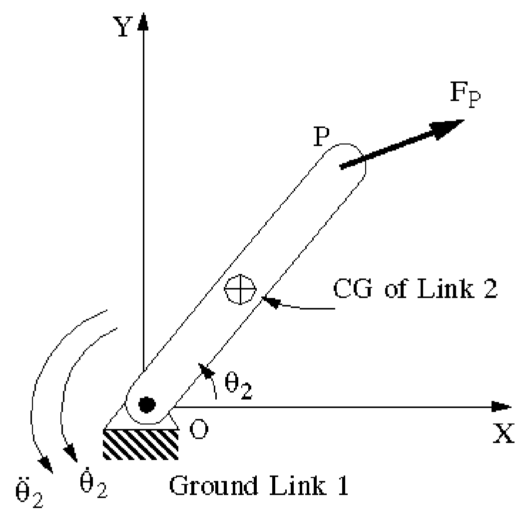

Figure 1 A single link in pure rotation about point O.

A rigid link (2) is pinned to the ground (1) at point O as shown in Figure 1. At some instant in time, the link is oriented at an angle, θ, with an angular velocity, $\dot{\theta}_\theta$, and angular acceleration, $\ddot{\theta}_\theta$. A known external force, $\mathbf{F}_P$ acts at point P. In order for this system to be in equilibrium, an external torque must be applied to the link and there must be some reaction forces at the pivot point. We can apply Newton's Laws of Motion to determine the torque that must be applied to the link and the reaction forces at the pin joint.

The free-body diagram and the equivalent dynamic system are shown in Figure 2. We have established a local coordinate system at the CG of the link. This coordinate system is attched to the moving link and its axes are parallel to the global coordinate system. In the free body diagram, $\mathbf{F}_{12}$ is the force of the ground (1) acting on link (2) and $\mathbf{T}_{12}$ is the torque that must be transmitted to the link in order to maintain the angular velocity and acceleration of the link. Notice the force and torque acting on the ground link (1). The force $\mathbf{F}_{21}$ is the force of the link acting on the ground and the torque, $\mathbf{T}_{21}$, is the torque transmitted from the link to the ground. The force $\mathbf{F}_{21}$ is equal in magnitude and opposite in direction to $\mathbf{F}_{12}$ (from Newton's Laws) and the $\mathbf{T}_{21}$ is equal and opposite to $\mathbf{T}_{12}$.

Newton's Laws also tell us that the sum of the external forces acting on the rotating link must be equal to the time rate of change of the momentum of the link. This allows us to write the vector equations:

$$\Sigma\bar{F} = \bar{F}_{\chi} + \bar{F}_{\Theta\theta} = m_{\theta}\bar{a}_{\Sigma\theta}$$
$$\Sigma\bar{T} = \bar{T}_{\Theta\theta} + (\bar{r}_{\Theta\theta} \times \bar{F}_{\Theta\theta}) + (\bar{r}_{\chi} \times \bar{F}_{\chi}) = I_{\Sigma\theta}\ddot{\theta}_{\theta}$$

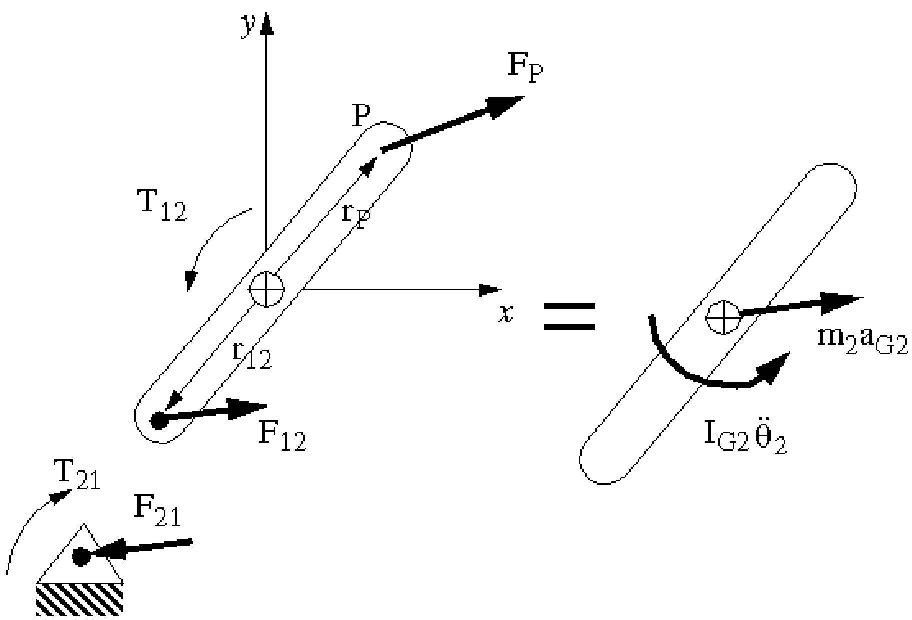

Figure 2 Free body diagram and equivalent dynamic model of the rotating link

In these expressions, m_2 is the mass of the link, $\mathbf{a}_{G2}$ is the acceleration of the CG of link 2, I_{G2} is the mass moment of inertia about an axis through the CG of link 2 and $\ddot{\theta}_{\theta}$ is the angular acceleration of link 2. The position vector $\mathbf{r}_{12}$ locates the pivot point with respect to the CG of link 2 and $\mathbf{r}_P$ locates the external force, $\mathbf{F}_P$, with respect to the CG of link 2. We can write these two vector equations as three scalar equations by taking the x and y components and expanding the cross products :

$$\Sigma F_{`} = F_{\chi`} + F_{\Theta\theta`} = m_{\theta} a_{\Sigma\theta`}$$
$$\Sigma F_{\hat{}} = F_{\chi\hat{}} + F_{\Theta\theta\hat{}} = m_{\theta} a_{\Sigma\theta\hat{}}$$
$$\Sigma T_{,} = T_{\Theta\theta} + r_{\Theta\theta`} F_{\Theta\theta\hat{}} - r_{\Theta\theta\hat{}} F_{\Theta\theta`} + r_{\chi`} F_{\chi\hat{}} - r_{\chi\hat{}} F_{\chi`} = I_{\Sigma\theta} \ddot{\theta}_{\theta}$$

The postion vectors, $\mathbf{r}_{12}$ and $\mathbf{r}_{p}$, the mass of the link, m_2, the moment of inertia, I_{G2} and the external force, $\mathbf{F}_{P}$ are all known quantities. We can calculate $\mathbf{a}_{G2}$ from $\mathbf{r}_{12}$, and $\dot{\theta}_{\theta}$ and $\ddot{\theta}_{\theta}$. In these expressions, the only unknown parameters are the components of $\mathbf{F}_{12}$ and the $\mathbf{T}_{12}$. We can solve these three linear equations for the three unknowns, F_{12x}, F_{12y} and T_{12}.